The Horizontal Geometry of Production Networks

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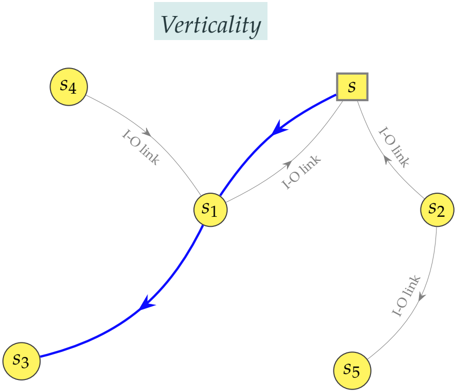
Background

- Comovement across sectors is an essential feature of contemporary economic systems
  - driven by independent sectoral shocks, not aggregate {*e.g.*, Gabaix (2011), Foerster *et al.* (2011)}

- Production networks $\longrightarrow$ role of Input-Output linkages {*e.g.*, Acemoglu *et al.* (2012)}
  - an idiosyncratic shock propagates to other sectors along several supply chains

→ Key mechanism: ***vertical complementarities***, through trade intensities {*e.g.*, Shea (2002)}
  - *linear economies* $\leftrightarrow$ comovement from even propagation of shocks {most of the literature};
  - *non-linear complementarities in production* $\leftrightarrow$ the even transmission of shocks is reversed, thereby allowing negative comovement {*e.g.*, Corsetti *et al.* (2008), Baqaee and Farhi (2019)}


Verticality
s_4
s
I-O link
I-O link
I-O link
s_1
s_2
I-O link
s_3
s_5

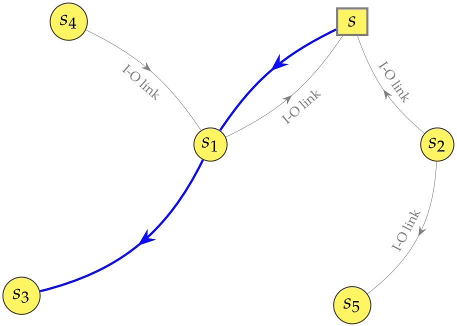
This paper: central intuition

- Beside the *vertical* dimension, there exists an ***horizontal*** geometry of a network
  - ☞ sectors are additionally held together by *demand and supply interdependencies from having in common a similar Input-Output structure*

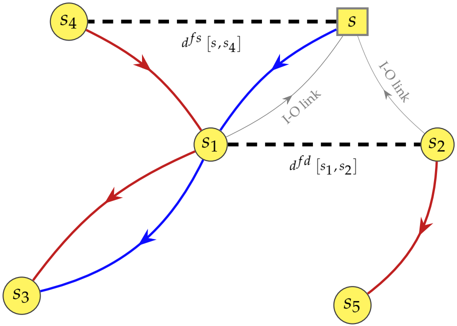
- Horizontality $\longrightarrow$ network "economic" distances between paired sectors
  - ***factor input demand*** distance, when *buying from* similar upstream sectors;
  - ***factor input supply*** distance, when *selling to* similar downstream sectors.

❊ Central result: ***horizontal complementarities balance the vertical transmission***
  - muted or negative comovement in nearby sectors, whereas distant sectors comove;
  - negative comovement is linearly induced $\rightarrow$ no need of micro-level substitution elasticities.

Verticality



Verticality + Horizontality

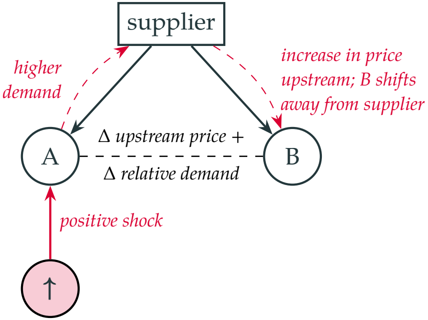


This paper: economic interpretation

- Intuition beyond network transmission:
  - *demand distance*: shocks trigger downstream buyer to shift away from upstream suppliers
    - ⋆ vertical and horizontal effects get mixed, making comovement ambiguous to interpret
  - *supply distance*: upstream shocks pass downstream, where inputs are used together
    - ⋆ horizontal propagation among suppliers is direct, mechanical, and unambiguous
- Core contrast:
  - *demand side*: mere increase in upstream price (no true horizontal complementarities);
  - *supply side*: substitution of upstream inputs through downstream demand reshuffling.

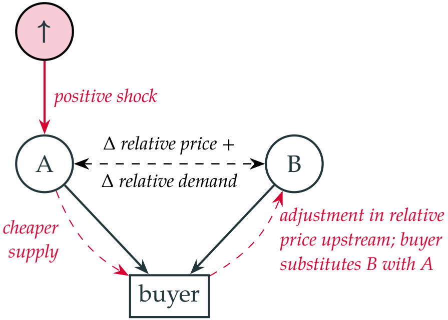
THIS PAPER: GRAPHING THE UNDERLYING ECONOMICS

Demand distance: upstream pass-through



downstream shifting away from upstream input
⇒ vertical price effect + horizontal ambiguity downstream

Supply distance: downstream pass-through



upstream inputs used jointly downstream
⇒ demand reallocation and clear horizontal transmission

This paper: overview of the results

Key logic

minor network economic distance $\Longleftrightarrow$ stronger horizontal transmission

- Empirical results:
  - demand-linked sectors: *muted/mixed comovement effects*;
  - closely supply-linked sectors: *negative comovement*;
  - farther supply-linked sectors: *positive comovement*.

- (linear) supply distance majorly disrupts standard vertical propagation
  - downstream effects across intermediate inputs are crucial to generate opposite responses
    {in line with non-linear theories in Atalay (2017) and Baqaee and Farhi (2019)}

What's to come

① Foundational theory, to define network distances and their role on comovement

- construct distance matrices in partial equilibrium
- sufficient statistics for distance-lead comovement under complementarities

② Network model, to determine how shocks propagate considering network distances

- general equilibrium effects on sectoral employment

③ Empirics, short-run *network effects* of sectoral variations to study theoretical results

- study the network effects of nearby sectors' employment changes on sectoral employment

④ Policy implications, for future novel perspectives (not today, but in the paper)

- improving the design of policy interventions through the network's horizontal geometry

Related literature

☞ Advance the understanding of how Input-Output economies work

- theory of production networks {*e.g.*, Long and Plosser (1983), Acemoglu *et al.* (2012), Carvalho (2014), Grassi (2017), Oberfield (2018), Liu (2019), Baqaee and Farhi (2019), Acemoglu and Azar (2020), vom Lehn and Winberry (2022), La'O and Tahbaz-Salehi (2022), Ghassibe (2024), Huo *et al.* (2025), Qiu *et al.* (2025), Antonova *et al.* (2025)}
  - wide range of topics (fiscal and monetary policy, efficiencies and inefficiencies, growth, financial frictions, international contexts) where the network is *vertical* only

  → ***Contribution***: first to introduce, explore, and formalize the network's *horizontal geometry*

- empirics on the functioning of Input-Output linkages {*e.g.*, Conley and Dupor (2003), Acemoglu *et al.* (2015), Barrott and Sauvagnat (2016), Ghassibe (2021), Barattieri and Cacciatore (2023), Barattieri *et al.* (2023)}

  → ***Contribution***: first to provide evidence of shocks' transmission due to network distances

- empirics on sectoral employment comovement {*e.g.*, Cooper and Haltiwanger (1990), Christiano and Fitzgerald (1998), Cassou and Vazquez (1990), Yedid-Levi (2016), Kim (2020)}

  → ***Contribution***: first to take a production network perspective

Foundational Theory

General framework

- Consider a general production function for sector $\{s, s', s'', \ldots, S\} \in \Phi(s)$

$$y(s) = z(s)\, f^{s}\left(n(s)\ ;\ \{x(s,s')\}_{s' \in \Phi(s)}\right)$$

- Combination of optimality conditions delivers

$$x(s,s') = f\left(\alpha(s,s'), p(s')\ ;\ \alpha(s), n(s), w(s)\right) \tag{1}$$

  - $\alpha(s,s')$: intensity of the good from sector-s' in the intermediates bundle used by sector-s

- Then, consider two types of *production network distances*:

① ***factor input demand***, when given sectors are *buying from* the same sector;

② ***factor input supply***, when given sectors are *selling to* the same sector.

‣ Regularity conditions ‣ Graph, distances ‣ Conditions for Lemmas

Factor input demand distance

- Substituting out eq. (1) for two sectors, say $\{s, s'\}$, actually *buying* their intermediate inputs from the same sector, say s^*, then total *log*-differentiation leads to

$$\underbrace{\left[\gamma_{s'}^{\mathcal{B}}\, d\log n\left(s'\right) - \delta_{s'}^{\mathcal{B}}\, d\log x\left(s', s^*\right)\right]}_{\Delta \text{ relative input share of sector-}s'} = \underbrace{\frac{\alpha\left(s, s^*\right)}{\alpha\left(s', s^*\right)}}_{d_{\leftarrow s^*}\left[s, s'\right]} \underbrace{\left[\gamma_{s}^{\mathcal{B}}\, d\log n\left(s\right) - \delta_{s}^{\mathcal{B}}\, d\log x\left(s, s^*\right)\right]}_{\Delta \text{ relative input share of sector-}s}$$

- Iteratively, the above condition can be written for all sectors from which the pair is buying. Stacked the outcome for all sectors, then

▸ Lemma, demand

$$\underset{S\times S}{\mathcal{F}} = \underset{S\times S}{\mathcal{D}^{fd}} \tag{2}$$

☞ In other words, the comovement of inputs of production across any pair of sectors is determined by their distance in the production network.

▸ Interpretation of matrix $\mathcal{D}$

Factor input supply distance

- Substituting out eq. (1) for two sectors, say $\{s, s'\}$, actually *selling* their intermediate inputs to the same sector, say s^*, then total *log*-differentiation leads to

$$\underbrace{\left[\gamma_{s'}^{\mathcal{Q}}\, d\log x\left(s^*, s\right) - \delta_{s'}^{\mathcal{Q}}\, d\log x\left(s^*, s'\right)\right]}_{\Delta \text{ relative quantity of intermediate inputs}} = \underbrace{\frac{\alpha\left(s^*, s\right)}{\alpha\left(s^*, s'\right)}}_{d_{\to s^*}\left[s, s'\right]} \underbrace{\left[\gamma_{s}^{\mathcal{P}}\, d\log p\left(s'\right) - \delta_{s}^{\mathcal{P}}\, d\log p\left(s\right)\right]}_{\Delta \text{ relative price of intermediate inputs}}$$

- Iteratively, the above condition can be written for all sectors to which the pair is selling. Stacked the outcome for all sectors, then

▸ Lemma, supply

$$\underset{S\times S}{\mathcal{R}} = \underset{S\times S}{\mathcal{D}^{fs}} \tag{3}$$

☞ In other words, the comovement across (marginal) revenues of production across any pair of sectors is determined by their distance in the production network.

Sufficient statistics

- So far intermediate inputs generally function as substitutes
  - intermediate inputs complementarity alters propagation {Atalay (2017), Baqaee and Farhi (2019)}

- Sufficient statistics provide reduced-form intuition on negative comovement ‣ Corollaries

$$\textit{demand-driven comovement}: \underbrace{\frac{\partial \log x(s,s^*)}{\partial \log x(s',s^*)}}_{<0} \propto -\sigma\left[1 - e(s,s^*)\right] \underbrace{\frac{\partial \log p(s^*)}{\partial \log x(s',s^*)}}_{>0}$$

$$\textit{supply-driven comovement}: \underbrace{\frac{\partial \log x(s^*,s)}{\partial \log x(s^*,s')}}_{<0} \propto \sigma\, e(s^*,s') \underbrace{\frac{\partial \log p(s')}{\partial \log x(s^*,s')}}_{<0}$$

☞ Network distances generate comovement as non-linear complementarities ‣ Interpretation

- as substitution elasticities matter in magnitude, not in the direction of comovement

Main theoretical takeaways

① *factor input demand* distance relates to changes in relative factors of production;

② *factor input supply* distance relates to changes in marginal revenues from trade;

③ the *distance network* connects sectors not just vertically, but also horizontally: ▸ Graph

→ introducing the *network distance* allows to consider ***also*** how sectors are related through *horizontal complementarities* in common demand and supply linkages . . .

. . . even if they are neither directly nor indirectly connected in the production network.

+

Different network distance relations complement a unique Input-Output structure (Theorem 1)

▸ Theorem 1 ▸ Stylized economies

Network Model

General framework

① Unit mass of households

$$\mathcal{U}_i\left(\{c_i(s)\}_{\forall s\in\Phi(s)}\,;\, n_i(s)\right) := \prod_{s\in\Phi(s)} c_i(s)^{\beta(s)} - \frac{n_i(s)^{1+\phi}}{1+\phi}$$

② Representative firm in each sector

$$y(s) = z(s)\left(n(s)\right)^{\alpha(s)} \prod_{s'\in\Phi(s)} \left(x\left(s,s'\right)\right)^{\alpha(s,s')} \tag{4}$$

③ Equilibrium in the labour market

$$n(s) = \left[\alpha\left(s\right)\ \frac{C}{c(s)}\ \beta\left(s\right)\ y(s)\right]^{\frac{1}{1+\phi}} \tag{5}$$

Incorporate network distances

- So far standard propagation mechanism $\rightarrow$ only ***vertical*** transmission

▸ Vertical effect

- To include network distances:

▸ Equilibrium conditions

  (i) general equilibrium manipulation of labour market equilibrium condition in eq. (A.4);

  (ii) insert intermediate results that derive Lemmas 1-2.

▸ Conditions for Lemmas

- In what follows I am going to discuss that the overall production network effects on sectoral comovement of employment can be summarized by

$$d\log \boldsymbol{N} = \Theta\Big\{d\log C + d\log \boldsymbol{\mathcal{S}} + d\log \boldsymbol{\mathcal{N}} + d\log \boldsymbol{\mathcal{D}}_{|j}\Big\} \quad \text{for} \quad j = \{fd, fs\}$$

→ *vertical* + ***horizontal*** dimensions

Distance-based propagation to sectoral employment

❊ Demand-driven distance responses of sectoral employment ▸ Proposition 3

$$d \log \boldsymbol{N} = \Theta \left\{ d \log C + d \log \boldsymbol{\mathcal{S}} + d \log \boldsymbol{\mathcal{N}} + \underbrace{\boldsymbol{\mathcal{D}}^{fd} \left[d \log \boldsymbol{N} \Big(\Phi \left(s \right) \Big) - d \log \boldsymbol{N} \right]}_{d \log \boldsymbol{\mathcal{D}}(n)} \right\} \tag{6}$$

❊ Supply-driven distance responses of sectoral employment ▸ Proposition 4

$$d \log \boldsymbol{N} = \Theta \left\{ d \log C + d \log \boldsymbol{\mathcal{S}} + d \log \boldsymbol{\mathcal{N}} + \underbrace{\boldsymbol{\mathcal{D}}^{fs} \left[d \log \boldsymbol{P} \Big(\Phi \left(s \right) \Big) - d \log \boldsymbol{P} \right]}_{d \log \boldsymbol{\mathcal{D}}(p)} \right\} \tag{7}$$

- Key driver of aggregate business cycles {*e.g.*, vom Lehn and Winberry (2022)} ▸ Business cycle
  - Δ aggregate GDP $\approx$ Δ sectoral productivities + Δ sectoral employment

Discussion

- Demand linkages make comovement ambiguous to interpret, while supply linkages make comovement transparent
  - overlap of vertical and horizontal transmission in *demand*;
  - direct competition for downstream markets (*i.e.*, changes in relative prices) in *supply*.

- In line with the non-linear theories in Atalay (2017) and Baqaee and Farhi (2019)
  - downstream complementarities are crucial ...

    ... and thus my *downstream demand pass-through* under supply distances ▸ Corollaries

- Finally, the horizontal geometry is:
  - stronger the higher is the number of network linkages; ▸ Stylized economies
  - not weighted by vertical dimension (Theorem 2) ▸ Theorem 2

Empirics

Data

- From the United States' Bureau of Economic Analysis (BEA) I collect information, for 3-digit U.S. 2017 NAICS sectors, on *employment* and on: ▸ I-O data ▸ I-O structure ▸ I-O Leontief
  - inter-sectoral linkages exploiting Input-Output (I-O) matrices:
    - ⋆ share of commodity production (*market share, after redefinitions* tables);
    - ⋆ I-O weights in producer values (*commodity-by-commodity total direct requirements* tables).
  - disregard small I-O transactions {*e.g.*, Acemoglu *et al.* (2012), Carvalho (2014)}
    - ⋆ no link if sector-s's total input sales/purchases are less than 1%
- *Unit*: 65 private sectors at 3-digit U.S. 2017 NAICS
- *Period*: 1998-2022
- *Network structure* at 2007

robust at different years

▸ Network representation, sales ▸ Network representation, purchases ▸ Distribution

Compute network distances

- Consider network distances at their *extensive margin*, $d(s, s') = \{1, 2, \ldots, d_{max}\}$
  - from intensive margin matrices of *Euclidean distances* between paired sectors ▸ Intensive margin

- Network distance-d computed as a ***shortest path problem*** ... ▸ Algorithm
  - algebraic version of standard algorithms exploiting matrix multiplication;
  - suitable for discrete-value matrices, efficient for small to medium-sized sparse graphs.

  ... that identifies how many steps are needed for a sector to reach all others ...

  ... exploiting *intensive margin distances* from the *directed* Input-Output network.

- Degree of separation between sectors within the network (low value, low distance)

▸ Graph ▸ Graph, characteristics

Employment changes in the network

① Identification, to extract exogenous dynamics {as in Barattieri and Cacciatore (2023)}

$$n_t(s) = \underbrace{\beta_y \, \dot{y}_t(s)}_{\text{output growth}} + \underbrace{\sum_d \beta_{y(d)} \, \dot{y}_t(\Phi_s, d)}_{\text{output growth, network distance}} + \underbrace{\sum_k \beta_{y(j)} \, \dot{y}_t(\Phi_s, k)}_{\text{output growth, upstream and downstream}}$$

$$+ \underbrace{\beta_{z(s)} \, \mathcal{Z}_t(s) + \beta_{\dot{\mathcal{Z}}(s)} \, \dot{\mathcal{Z}}_t(s)}_{\text{sector-level controls}} + \underbrace{\beta_z \, \mathcal{Z}_t + \beta_{\dot{\mathcal{Z}}} \, \dot{\mathcal{Z}}_t}_{\text{aggregate controls}} + \underbrace{\psi(s)}_{\text{sector Fe}} + u_t^n(s)$$

- estimated residuals $\left(\widehat{u}_t^n(s)\right)$ identify the residual variation in sectoral employment

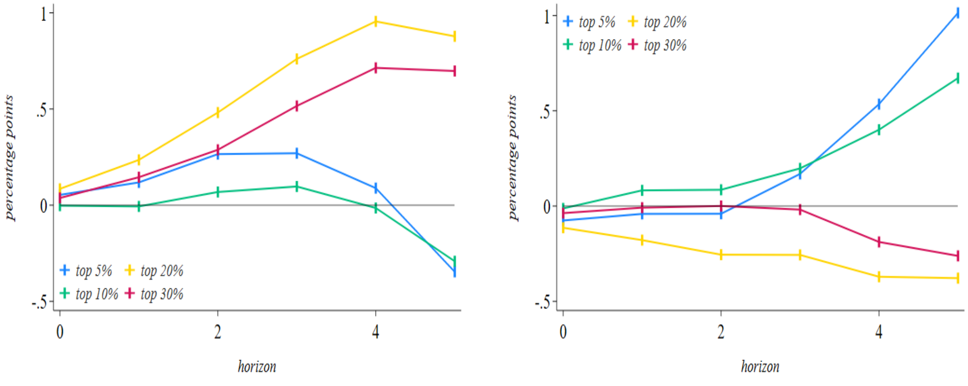
② Local Projections, for short-run dynamics response {as in Barattieri and Cacciatore (2023)}

$$\dot{n}_{t+\hbar}^{cum}(s) = \beta_{\widehat{n}(d),\hbar} \sum_{s' \neq s} \widehat{u}_t^n(s', d) + \underbrace{\psi(t+\hbar) + \psi_\hbar(s)}_{\text{horizon and sector Fe}} + v_{t+\hbar}^n(s, d)$$

Variables' definition

Employment effects of most connected sectors, distance

factor input demand and directed network



(a) $d = 1$, *directed*

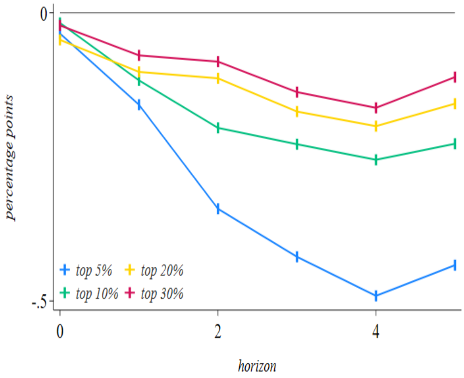
(b) $d = 2$, *directed*

Figure 3: Demand linkages and comovement in interlinked sectors

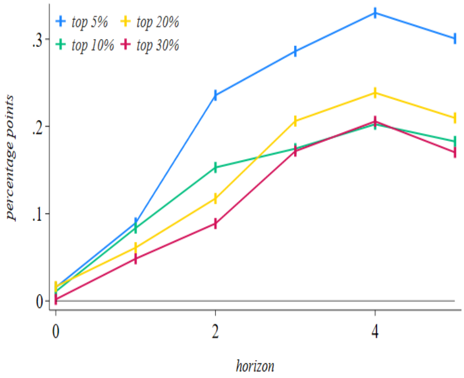
▸ Average, direct, demand

Employment effects of most connected sectors, distance

factor input supply and directed network



(a) $d = 1$, *directed*



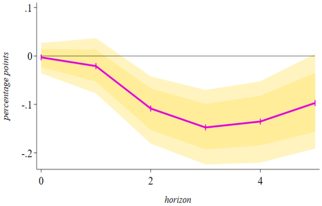
(b) $d = 2$, *directed*

Figure 4: Supply linkages and comovement in interlinked sectors

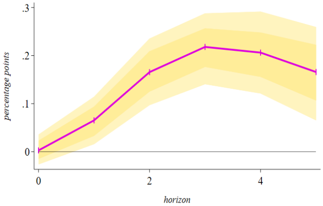
Average, direct, supply

Employment effects in most peripheral sectors, distance

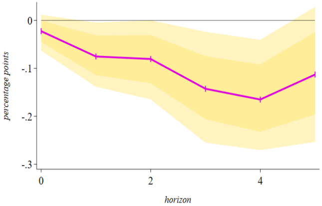
factor input demand, factor input supply and directed network



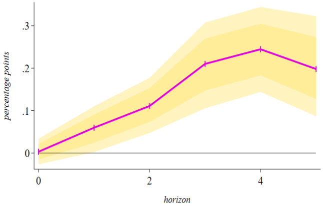
(a) $d = 1$, demand



(b) $d = 2$, demand



(c) $d = 1$, supply



(d) $d = 2$, supply

Policy implications

Novel perspectives: policy design

☞ Horizontal complementarities are crucial for the transmission of sectoral variations

- ***Fiscal policy***, typically generating upstream transmission { *e.g.*, Barattieri *et al.* (2025)};
  - it works by expanding sector-specific demand, and thus the supply of others;

  → thus, horizontal geometry is at the core of the effects from government spending.
  - targeted government spending, fiscal multiplier, tariff policy?

- ***Industrial policy***, characterised by two crucial results:
  - market imperfections accumulate through backward demand linkages {Liu (2019)}, and positive effects on buyers of targeted sectors {Lane (2025)};

  → how do interventions affect untargeted sectors exhibiting similar Input-Output structure?

- ***Transmission mechanism***, role of (consumption or production) complementarities
  - asymmetric supply shock is a demand-like shock at the aggregate level {Guerrieri *et al.* (2022)};

  → horizontal complementarities may clearly, easily and linearly identify demand and supply forces within the network that guide such Keynesian transmission.

Novel perspectives: theoretical insights

- On the modelling side, expanded framework for:
  - global production networks, extending beyond vertical supply chains {*e.g.*, Huo *et al.* (2025)};
  - firm-to-firm networks, characterized by high sparsity {*e.g.*, Bohem *et al.* (2019)};
  - endogenous network formation and inputs specificity {*e.g.*, Carvalho and Voigtländer (2015)};
  - complementarity and sectoral shocks to aggregate outcomes {*e.g.*, Atalay (2017)}.

- Horizontal dimension is suited for broader class of interdependencies, including investment {*e.g.*, vom Lehn and Winberry (2022)} and financial {*e.g.*, Acemoglu *et al.* (2015), Huremovic *et al.* (2025)} networks

Conclusion

Take-away remarks

- *Horizontal dimension* of a network complements the vertical (Leontief) dimension
  - ☞ it is not the mere existence of Input-Output connections, but **their demand-supply geometry that critically shapes the transmission of micro-originated shocks**

- Horizontal geometry captured by *network "economic" distances* between sectors
  - demand-based distances work through production inputs dynamics, while supply-based distances induce negative comovement due to revenue reallocation;
  - → supply distances are crucial to generate opposite responses, and they linearly capture non-linear downstream complementarities effects. {*e.g.*, Atalay (2017), Baqaee and Farhi (2019)}

- Comprehensive framework to build distance matrices and form theoretical insights
  - test and corroborate model predictions on sectoral U.S. Input-Output data;
  - suitable for analysis involving the network structure and to design policy prescriptions.

Appendix

This paper: key findings

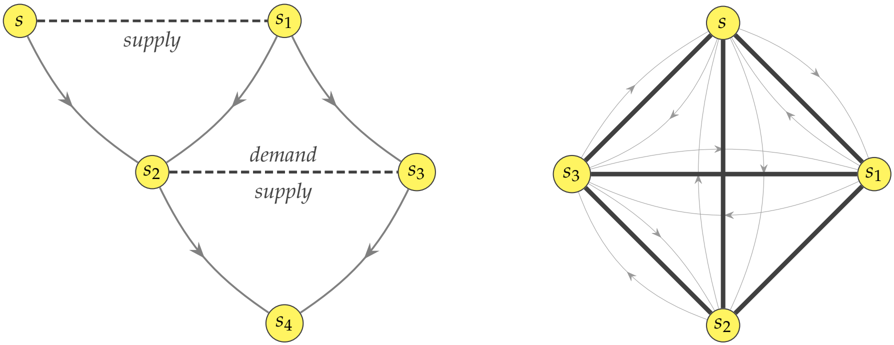
- ***Theoretical results***, from partial to general equilibrium
  - *demand-driven* distance measures comovement in inputs of production, whereas *supply-driven* distance captures the reallocation dynamics of intermediate goods;
    - ⋆ *Leontief* transmission is mitigated when sectors are close;
  - network distances connect sectors otherwise un-connected (Theorem 1);
  - network distances are not themselves weighted by Input-Output structure (Theorem 2).
- ***Empirical results***, using U.S. Input-Output data
  - demand-lead complementarities induce ambiguous employment comovement;
  - supply-lead complementarities induce opposite employment comovement;
  - when distance values are high (further sectors), there is always positive comovement.

Assumption 1 (Production technology requirements)
As main characteristics of a production function: (i) it has constant returns to scale in either $x\big(s, \Phi(s)\big)$ *and* $n(s)$*, so that production inputs shares sum to one,* $\alpha(s) + \sum_{s'} \alpha(s, s') = 1$*; (ii) it is differentiable, continuous, strictly quasi-concave, homogeneous of degree one, and increasing in* $z(s)$*,* $x\big(s, \Phi(s)\big)$*, and* $n(s)$*; (iii) the case* $f(0\,;\,\cdot)$ *is ruled out as labour input is essential to production; (iv) at least two elements of* $x\big(s, \Phi(s)\big)$ *has to be positive, thus* $f(\cdot\,;\,0)$ *cannot exist; finally, (v) part of the sectoral output is directly produced by the sector,* $x(s, s) > 0\ \forall\ s$*, that is production with a bundle of intermediate inputs as* $f\big(\cdot\,;\,x(s, \Phi_s)\big)$ *is precluded.*

- Note that any functional form chosen for the production function of eq. (4) must satisfy the above regularity conditions

Go back

Stylized representation of distances



(a) Distance definition

(b) Distance network

Figure A.1: Horizontal Input-Output geometries

Go back

Intermediate conditions in partial equilibrium

- Solving the system for *factor input demand* network distance

$$\begin{cases} x\left(s,s^*\right) = \alpha\left(s,s^*\right) w\left(s\right) n\left(s\right)\ \alpha\left(s\right)^{-1} \dfrac{1}{p\left(s^*\right)} \\ x\left(s',s^*\right) = \alpha\left(s',s^*\right) w\left(s'\right) n\left(s'\right)\ \alpha\left(s'\right)^{-1} \dfrac{1}{p\left(s^*\right)} \end{cases}$$

would deliver the condition $\frac{n(s')}{x(s',s^*)} = \frac{\alpha(s,s^*)}{\alpha(s',s^*)} \frac{n(s)}{x(s,s^*)}$.

- Solving the system for *factor input sypply* network distance

$$\begin{cases} x\left(s^*,s\right) = \alpha\left(s^*,s\right) w\left(s^*\right) n\left(s^*\right)\ \alpha\left(s^*\right)^{-1} \dfrac{1}{p\left(s\right)} \\ x\left(s^*,s'\right) = \alpha\left(s^*,s'\right) w\left(s^*\right) n\left(s^*\right)\ \alpha\left(s^*\right)^{-1} \dfrac{1}{p\left(s'\right)} \end{cases}$$

would deliver the condition $\frac{x(s^*,s)}{x(s^*,s')} = \frac{\alpha(s^*,s)}{\alpha(s^*,s')} \frac{p(s')}{p(s)}$.

Go back, framework | Go back, model | Go back, employment equilibrium conditions under distances

Lemma 1 (Factor input demand)

Changes in the ratio of production inputs' quantities across any pair of sectors buying from the same sector(s) is driven by their ratio of intermediate intensities with whom the pair is purchasing from

$$\underset{S\times S}{\boldsymbol{\mathcal{F}}} = \underset{S\times S}{\boldsymbol{\mathcal{D}}^{fd}} \tag{A.1}$$

Lemma 2 (Factor input supply)

Changes in the ratio of intermediate input profits across any pair of sectors selling to the same sector(s) is driven by their ratio of intermediate intensities with whom the pair is purchasing from

$$\underset{S\times S}{\boldsymbol{\mathcal{R}}} = \underset{S\times S}{\boldsymbol{\mathcal{D}}^{fs}} \tag{A.2}$$

▸ Demand distance ▸ Supply distance

Interpretation of matrix $\mathcal{D}$

- On the theoretical side, each cell identifies "network proximity"
  - the higher $d[s, s']$, the larger the transmission $\longleftrightarrow$ stronger horizontal complementarity

- Once taken to the data, each cell measures "network distance", computed as:
  - *Euclidean distance*, $d_{\leftarrow s^*}[s, s'] = \left[\alpha(s, s^*) - \alpha(s', s^*)\right]^2$; or
  - *shortest path algorithm* (how many steps are necessary for a sector to reach another one).

- Some characteristics of the matrix (Theorem 1):
  - non-negative and full, symmetric outside the main diagonal, with $diag(\mathcal{D}) = 0$, and it is one for each distance measure

Theorem 1

- Horizontal dimension is not weighted by vertical dimension (Theorem 2)

Theorem 2

Go back

Matrix configuration

The *network distance matrix* is defined as

$$\underset{S\times S}{\mathcal{D}} = \begin{bmatrix} \left(\frac{\alpha(s,s)}{\alpha(s,s)},\ldots,\frac{\alpha(s,S)}{\alpha(s,S)}\right) & \left(\frac{\alpha(s,s)}{\alpha(s',s)},\ldots,\frac{\alpha(s,S)}{\alpha(s',S)}\right) & \cdots & \left(\frac{\alpha(s,s)}{\alpha(S,s)},\ldots,\frac{\alpha(s,S)}{\alpha(S,S)}\right) \\ \left(\frac{\alpha(s',s)}{\alpha(s,s)},\ldots,\frac{\alpha(s',S)}{\alpha(s,S)}\right) & \left(\frac{\alpha(s',s)}{\alpha(s',s)},\ldots,\frac{\alpha(s',S)}{\alpha(s',S)}\right) & \cdots & \left(\frac{\alpha(s',s)}{\alpha(S,s)},\ldots,\frac{\alpha(s',S)}{\alpha(S,S)}\right) \\ \vdots & \vdots & \ddots & \vdots \\ \left(\frac{\alpha(S,s)}{\alpha(s,s)},\ldots,\frac{\alpha(S,S)}{\alpha(s,S)}\right) & \left(\frac{\alpha(S,s)}{\alpha(s',s)},\ldots,\frac{\alpha(S,S)}{\alpha(s',S)}\right) & \cdots & \left(\frac{\alpha(S,s)}{\alpha(S,s)},\ldots,\frac{\alpha(S,S)}{\alpha(S,S)}\right) \end{bmatrix} = \left[d\left(s,s'\right)\right]$$

▸ Go back

Sufficient statistics for distance-lead comovement

Corollary 1 (Sufficient statistics for demand-driven comovement)

In the single-input case, the sufficient statistic for negative comovement in demand implies $\delta_{\leftarrow s^*}[s, s'] \propto -\sigma\, \tau^{fd}$, *with the response largely determined by the upstream pass-through* τ^{fd}. *In the multi-input CES case, the sufficient statistics augments to*

$$\delta_{\leftarrow s^*}[s, s'] \;\propto\; -\sigma \left[1 - e\left(s, s^*\right)\right] \tau^{fd}$$

with relative intermediate inputs changes from common supplier, $\delta_{\leftarrow s^*}[s, s'] = \frac{\partial \log x(s,s^*)}{\partial \log x(s',s^*)}$, *jointly determined by substitution elasticity, downstream sectoral expenditure share, and upstream pass-through. Negative comovement occurs if* $\tau^{fd} > 0$.

Corollary 2 (Sufficient statistics for supply-driven comovement)

In the single-input case, no sufficient statistic exists for negative comovement in supply due to the absence of competition for downstream markets. Differently, in the multi-input CES case, the sufficient statistic is given by

$$\delta_{\rightarrow s^*}[s, s'] \;\propto\; \sigma\, e\left(s^*, s'\right)\, \tau^{fs}$$

with relative intermediate inputs changes to common buyer, $\delta_{\rightarrow s^*}[s, s'] = \frac{\partial \log x(s^*,s)}{\partial \log x(s^*,s')}$, *jointly determined by substitution elasticity, sectoral revenues for upstream supplier, and downstream pass-through. Negative comovement occurs if* $\tau^{fs} < 0$.

▸ Go back, sufficient statistics ▸ Go back, model discussion ▸ Interpretation

Interpretation of the sufficient statistics

- ***Demand-based network distances***
  - capture how easily a downstream buyer can switch among suppliers, without creating "true" horizontal linkages $\longleftrightarrow$ *upstream pass-through effect*;
  - reflect substitution patterns across buyers sharing common upstream suppliers;
  - ambiguous propagation effects, as they lack genuine complementarities among buyers.

- ***Supply-based network distances***
  - generate genuine horizontal complementarities among sectors whose outputs are jointly used by common downstream buyers $\longleftrightarrow$ *downstream pass-through effect*.
  - shocks to one supplier propagate horizontally as other suppliers adjust production due to downstream complementarities;
  - strong comovement effects, consistent with non-linear network theories emphasizing complementarities in intermediate inputs. {*e.g.*, Atalay (2017), Baqaee and Farhi (2019)}

▸ Go back, sufficient statistics ▸ Go back, corollaries

Theorem 1 (On the configuration of network "economic" distances)

Let the set of sectors being $\Phi(s)$, and $\alpha^j(\cdot)$, for $j = \{fd, fs,\}$, the normalized Input-Output coefficients. Then, demand- and supply-based distance matrices satisfy: (i) for any $s, s' \in \Phi(s)$ there exists a common neighbour-s^*, so that both $\mathcal{D}^{fd}$ and $\mathcal{D}^{fs}$ are non-negative and full; (ii) a given Input-Output structure uniquely determines $\mathcal{D}^{fd}$ and $\mathcal{D}^{fs}$, which differ entrywise unless the corresponding bilateral trade with the common neighbour coincides; and (iii) if $\alpha^j(\cdot)$'s are non-symmetric, then all entries of $\mathcal{D}^{fd}$ and $\mathcal{D}^{fs}$ differ; conversely, if $S \geq 4$, then both share at least one identical entry whenever at least $S-2$ sectors trade symmetrically with a common sector and at least one additional symmetric trade relation elsewhere in the network exists.

▸ Go back, distance matrix characteristics ▸ Go back, theory recap

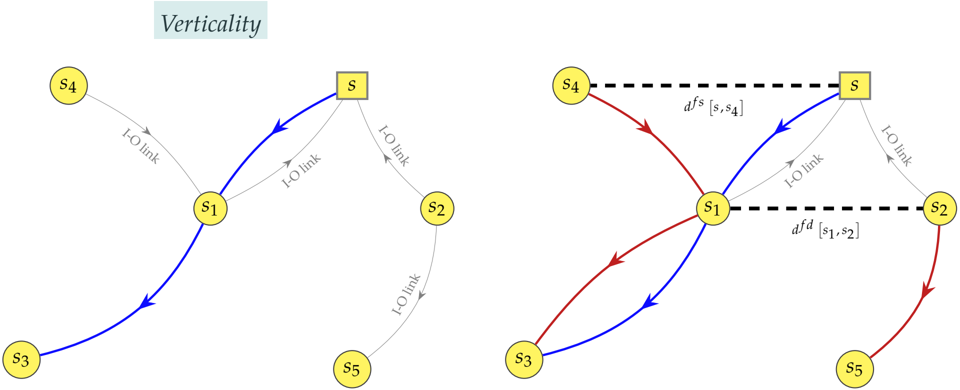
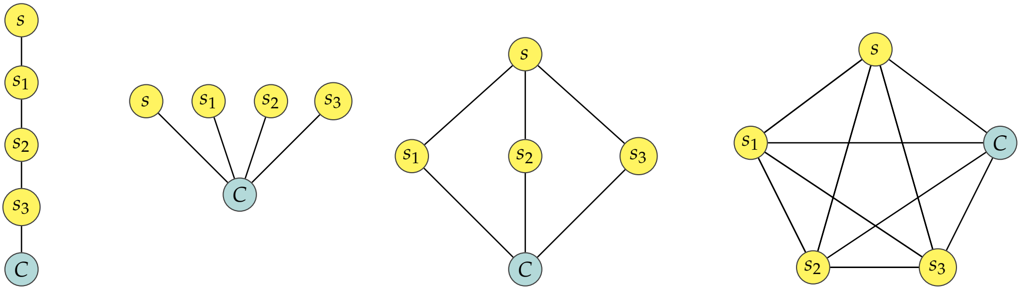


Figure A.2: Verticality and horizontality of a transmission

Go back

Stylized economies



(a) Sequential (b) Rhomboid (c) Star

Figure A.3: Benchmark economies

Go back, theory recap
Go back, model discussion

Modelling the demand side

- Each household $i \in \mathcal{I}$ has the following period-utility function

$$\mathcal{U}_i := f\left[c_i\left(s\right), c_i\left(s'\right), \ldots, c_i\left(S\right)\ ;\ n_i(s)\right]$$

$$:= \prod_{s \in \Phi(s)} c_i(s)^{\beta(s)} - \frac{n_i(s)^{1+\phi}}{1+\phi} \qquad \textit{with} \quad \sum_{s \in \Phi(s)} \beta\left(s\right) = 1$$

and the budget constraint $\sum_{s \in \Phi(s)} p(s) c_i\left(s\right) = w(s)\, n_i(s) + \sum_{s \in \Phi(s)} D_i(s)$.

- Optimality conditions, once aggregated across households, are given by

$$\frac{\partial \mathcal{L}}{\partial c_i(s)} \quad : \quad \frac{p(s)c(s)}{\beta\left(s\right)} = \frac{p\left(s'\right)c\left(s'\right)}{\beta\left(s'\right)}$$

$$\frac{\partial \mathcal{L}}{\partial n_i(s)} \quad : \quad w(s) = n(s)^{\phi} p(s) c(s) \left[\beta\left(s\right)\ C\right]^{-1}$$

Go back

Modelling the supply side

- Production function in sector-$s \in \Phi(s)$ is Cobb-Douglas

$$y(s) = z(s) f^s\left(n(s) \; ; \; \{x(s,s')\}_{s' \in \Phi(s)}\right)$$

$$= z(s)\left(n(s)\right)^{\alpha(s)} \prod_{s' \in \Phi(s)} \left(x(s,s')\right)^{\alpha(s,s')} \qquad \textit{with} \quad \alpha(s) + \sum_{s' \in \Phi(s)} \alpha(s,s') = 1$$

with profits $D(s) = p(s)\, y(s)$, and costs $\mathcal{C}(s) = w(s)\, n(s) + \sum_{s' \in \Phi(s)} p(s')\; x(s,s')$

- Optimality conditions are given by

$$\frac{\partial \mathcal{L}}{\partial n(s)} \quad : \quad n(s) = \alpha(s)\left[p(s)\, y(s)\right]\left(w(s)\right)^{-1}$$

$$\frac{\partial \mathcal{L}}{\partial x(s,s')} \quad : \quad x(s,s') = \alpha(s,s')\left[p(s)\, y(s)\right]\left(p(s')\right)^{-1}$$

Go back

Aggregation

① Labour market

- sectoral labour demand: $n^{dem}(s) = \left[\frac{\beta(s)C}{p(s)c(s)} w(s)\right]^{\frac{1}{\phi}}$;
- sectoral labour supply: $n^{sup}(s) = \alpha(s)\left[p(s)\,y(s)\right]\left(w(s)\right)^{-1}$;
- aggregate equilibrium: $\sum_s n^{dem}(s) = \sum_s n^{sup}(s) \quad \longleftrightarrow \quad N \equiv N^{dem} = N^{sup}$.

② Good market

- start by $\sum_s y(s) = \int_i \left(\sum_s c_i(s)\right)\mathrm{d}i + \sum_s \left(\sum_{s'} x(s,s')\right)$ which, ...

  ... once aggregated, yields the equilibrium condition in the good market: $Y = C + X$.

③ Aggregate resource constraint

- by aggregating the individual household's budget constraint both across households and sectors, $P^c \int_i C_i \mathrm{d}i = \sum_s \left(w(s) \int_i n_i(s)\mathrm{d}i\right) + \sum_s \left(\int_i \mathcal{D}_i(s)\mathrm{d}i\right)$, ...

  ... then, the economy aggregate resource constraint is just $P^c C = WN + \mathcal{D}$.

▸ Go back

Equilibrium conditions

A competitive equilibrium for this efficient economy is defined by a set of sectoral prices for labour and intermediate inputs, $\{w(s), p(s)\}_{s\in\Phi(s)}$, *a set of sectoral production input quantities,* $\{n(s), x(s,s')\}_{s,s'\in\Phi(s)}$, *an exogenous sectoral productivity,* $\{z(s)\}_{s\in\Phi(s)}$, *and a set of aggregate quantities,* $\Omega = (Y, C, N, X, D)$, *such that*

(a) *each household satisfies its optimality conditions;*

(b) *representative firm in each sector maximizes profits;*

(c) *all markets clear, shaping* $\Omega(\cdot)$.

Go back

Proposition 1 (First-order propagation for I-O inputs)

Consider a market economy characterized by Input-Output linkages as stated in eq. (4). Then, the response of sectoral employment to changes in the set of sectoral intermediate inputs is a first-order (log-linear) approximation given by

$$d\log \boldsymbol{N} = \Theta \left\{ d\log C + \underbrace{d\log \boldsymbol{z} - d\log \boldsymbol{C}}_{d\log \boldsymbol{S}} + \boldsymbol{H}\, d\log \boldsymbol{X} \right\} \tag{A.3}$$

where:

(a) $\boldsymbol{N}$ *identifies sectoral employment levels;*
(b) $\Theta = \frac{1}{1+\phi+\boldsymbol{\alpha}}$ *is a compound of structural parameters, and* $\boldsymbol{\alpha} = \left[\alpha(s), \ldots, \alpha(S)\right]$;
(c) C *is aggregate consumption;*
(d) $\boldsymbol{S}$ *identifies sector-specific changes in productivity and consumption;*
(e) $\boldsymbol{H}$ *is the Input-Output matrix, comprising sectoral weight on other sectors, influencing the elements of* $\boldsymbol{X}$*, a matrix with intermediate inputs purchased by a given sector from each of the other sectors.*

- Intermediate inputs bought by sector-s can be taught as $x(s,s') = \vartheta(s,s') \; y(s')$

- Thus, eq. (5) can be rewritten as

$$n(s) = \left[\alpha(s) \; \frac{C}{c(s)} \beta(s) \;\; z(s) \Big(n(s)\Big)^{\alpha(s)} \prod_{s' \neq s} \Big(\vartheta(s,s') \, y(s')\Big)^{\alpha(s,s')}\right]^{\frac{1}{1+\phi}} \tag{A.4}$$

→ sectoral employment levels relation through the production network structure

❇ First-order responses of sectoral employments

▸ Proposition 2

$$d\log \boldsymbol{N} = \Theta \left\{ d\log C + d\log \boldsymbol{\mathcal{S}} + \underbrace{\boldsymbol{H}\Big(\Psi_{s,s'}\Big)\Big[d\log \boldsymbol{z} + \boldsymbol{\alpha}\; d\log \boldsymbol{N}\Big] + \boldsymbol{\mathcal{E}}\; d\log \boldsymbol{X}}_{d\log \boldsymbol{\mathcal{N}}} \right\} \tag{A.5}$$

▸ Go back

Proposition 2 (First-order propagation for employment)

Consider an Input-Output economy defined by a labour market equilibrium as stated in eq. (A.4). Then, the response of sectoral employment to other sectors' employment is a first-order (log-linear) approximation given by

$$d\log \boldsymbol{N} = \Theta \left\{ d\log C + d\log \boldsymbol{\mathcal{S}} + \underbrace{\boldsymbol{H}\left(\Psi_{s,s'}\right)\left[d\log \boldsymbol{z} + \boldsymbol{\alpha}\, d\log \boldsymbol{N}\right] + \boldsymbol{\mathcal{E}}\, d\log \boldsymbol{X}}_{d\log \boldsymbol{\mathcal{N}}} \right\} \tag{A.6}$$

where:

(a) $\boldsymbol{N}$ *identifies sectoral employment levels;*

...

(e) $\boldsymbol{\mathcal{N}}$ *identifies the production network effect of other sectors' changes in productivities, employment levels, and intermediate inputs usage impacting sector-s where:*

- ⋆ $\boldsymbol{H}(\Psi_{s,s'})$ *is the Input-Output matrix, comprising the weight of each sector on other sectors, whose entries are set to 0 whenever* $s = s'$;
- ⋆ $\boldsymbol{\mathcal{E}} = \boldsymbol{H}(\Psi_{s,s'})'\boldsymbol{H}(\Psi_{s',s})$ *is a compounded network effect, made of the inner product of the Input-Output matrix, influencing the elements of* $\boldsymbol{X}$, *a matrix with intermediate shares purchased by a given sector from each of the other sectors.*

▸ Go back ▸ Components ▸ Leontief inverse

Equilibrium employment under network distances

- Once inserting the production function of eq. (4) in labour market equilibrium condition in eq. (A.4), its manipulation in general equilibrium would result in

① ***factor input demand distance*** where, then, it should be inserted Lemma 1

$$n(s) = \left[\alpha(s) \ \frac{C}{c(s)} \ \beta(s) \ z(s) \ n(s)^{\alpha(s)} \ \aleph^{fd} \ \Lambda^{fd} \ \underbrace{\prod_{s' \in \Phi(s)} \left[\prod_{s \in \Phi(s)} \left(\frac{x(s', s)}{x(s, s)} \right)^{\frac{\alpha(s', s)}{\alpha(s, s)}} \right]}_{\Xi^{fd}} \right]^{\frac{1}{1+\phi}} \tag{A.7}$$

② ***factor input supply distance*** where, then, it should be inserted Lemma 2

$$n(s) = \left[\alpha(s) \ \frac{C}{c(s)} \ \beta(s) \ z(s) \ n(s)^{\alpha(s)} \ \aleph^{fs} \ \Lambda^{fs} \ \underbrace{\prod_{s' \in \Phi(s)} \left[\prod_{s \in \Phi(s)} \left(\frac{x(s', s)}{x(s', s')} \right)^{\frac{\alpha(s', s)}{\alpha(s', s')}} \right]}_{\Xi^{fs}} \right]^{\frac{1}{1+\phi}} \tag{A.8}$$

Defining components for network distances

- (fd) $\diamond$ $\aleph^{fd} = \prod_{s' \in \Phi(s)} \left(\prod_{s \in \Phi(s)} x\left(s', s\right)^{\alpha(s',s)} \right)^{\frac{\alpha(s,s')}{1+\phi}}$ identifies the set of intermediate inputs bought by sector-s and all the other sectors connected to it;
- (fd) $\diamond$ $\boldsymbol{\Lambda}^{fd} = \prod_{s' \in \Phi(s)} \left(\vartheta\left(s, s'\right) \; z(s') \; n(s')^{\alpha(s)} \right)^{\alpha(s,s')}$ captures the relevance of other sectors' productivities and employment levels on sector-s through the production network;
- (fd) $\diamond$ $\boldsymbol{\Xi}^{fd}$ determines the ratio among the intermediate inputs between sector-s and each of the other sectors, when these are buying from the same upstream sector.

- (fs) $\diamond$ $\aleph^{fs} = \prod_{s' \in \Phi(s)} \left(\prod_{s' \in \Phi(s)} x\left(s', s'\right)^{\alpha(s',s')} \right)^{\frac{\alpha(s,s')}{1+\phi}}$ is the compounded set of intermediate inputs;
- (fs) $\diamond$ $\boldsymbol{\Lambda}^{fs} = \prod_{s' \in \Phi(s)} \left(\vartheta\left(s, s'\right) \; z(s') \; n(s')^{\alpha(s)} \right)^{\alpha(s,s')}$ considers sectoral productivities and employment levels;
- (fs) $\diamond$ $\boldsymbol{\Xi}^{fs}$ determines the ratio among the intermediate inputs of paired sectors, when these are selling to the same downstream sector.

Go back

Proposition 3 (First-order propagation for employment under demand distance)

Consider an Input-Output economy defined by a labour market equilibrium as stated in eq. (A.7). Then, the response of sectoral employment to other sectors' employment is a first-order (log-linear) approximation given by

$$d\log \mathbf{N} = \Theta \left\{ d\log C + d\log \boldsymbol{\mathcal{S}} + d\log \boldsymbol{\mathcal{N}} + \underbrace{\boldsymbol{\mathcal{D}}^{fd} \left[d\log \mathbf{N}\Big(\Phi\left(s\right)\Big) - d\log \mathbf{N} \right]}_{d\log \boldsymbol{\mathcal{D}}(n)} \right\} \tag{A.9}$$

where:

(a) $\mathbf{N}$ *identifies sectoral employment levels;*

...

(f) $\boldsymbol{\mathcal{D}}^{fd}$ *identifies all the demand-based distances across any pair of sectors;*

(g.i) $\boldsymbol{\mathcal{D}}(n)$ *identifies the production network distance effect of other sectors' variations in employment levels impacting sector-s, when these are buying their intermediate inputs from the same sector(s).*

Go back

Components Leontief inverse

PROPOSITION 4 (First-order propagation for employment under supply distance)

Consider an Input-Output economy defined by a labour market equilibrium as stated in eq. (A.8). Then, the response of sectoral employment to other sectors' employment is a first-order (log-linear) approximation given by

$$d\log \boldsymbol{N} = \Theta \left\{ d\log C + d\log \boldsymbol{\mathcal{S}} + d\log \boldsymbol{\mathcal{N}} + \underbrace{\boldsymbol{\mathcal{D}}^{fs}\left[d\log \boldsymbol{P}\Big(\Phi\left(s\right)\Big) - d\log \boldsymbol{P} \right]}_{d\log \boldsymbol{\mathcal{D}}(p)} \right\} \tag{A.10}$$

where:

(*a*) $\boldsymbol{N}$ *identifies sectoral employment levels;*

...

(*f*) $\boldsymbol{\mathcal{D}}^{fs}$ *identifies all the supply-based distances across any pair of sectors;*

(*g.ii*) $\boldsymbol{\mathcal{D}}(p)$ *identifies the production network distance effect of other sectors' variations in employment levels impacting sector-s, when these are selling part of their output to the same sector(s).*

▸ Go back ▸ Components ▸ Leontief inverse

Defining components for propagation

Given $\Psi_{s,s'}$ being the set of sectors whose I-O weight is set to zero whenever $s = s'$:

- $\underset{S\times 1}{\boldsymbol{N}} = \left[n(1),\ldots,n(s),\ldots,n(S)\right]$;
- $\Theta = \frac{1}{1+\phi+\boldsymbol{\alpha}}$, where ϕ is a scalar for labour share and inverse Frisch elasticity;
- $d\log \boldsymbol{\mathcal{S}} := d\log \underset{S\times 1}{\boldsymbol{z}} - d\log \underset{S\times 1}{\boldsymbol{C}}$;
- $d\log \boldsymbol{\mathcal{N}} := \underset{S\times S}{\boldsymbol{H}}\left(\Psi_{s,s'}\right)\left\{d\log \underset{S\times 1}{\boldsymbol{z}} + \boldsymbol{\alpha}\, d\log \underset{S\times 1}{\boldsymbol{N}}\right\} + \underset{S\times S}{\boldsymbol{\mathcal{E}}}\, d\log \underset{S\times S}{\boldsymbol{X}}$, where:

  (a) $\underset{S\times S}{\boldsymbol{\mathcal{E}}} = \underset{S\times S}{\boldsymbol{H}}\left(\Psi_{s,s'}\right)' \underset{S\times S}{\boldsymbol{H}}\left(\Psi_{s',s}\right)$ is a compounded network effect, made of the inner product of the Input-Output matrix whose entries are set to 0 whenever $s = s'$;

  (b) $\underset{S\times S}{\boldsymbol{X}}$ is a matrix identifying the set of intermediate inputs purchased by sector-s from other sectors, taken as a share of other sectors' production.

Go back, I-O inputs | Go back, employment | Go back, demand distance | Go back, supply distance

Proposition 5 (First-order *Leontief inverse* propagation for employment)

Consider an Input-Output economy defined by a labour market equilibrium as stated in eq. (A.4). Then, the direct and indirect network effects governing the response of sectoral employment is a first-order (log-linear) approximation given by

$$d\log \boldsymbol{N} = \Theta \left\{ d\log C + d\log \boldsymbol{\mathcal{S}} + \underbrace{\boldsymbol{H}\left(\Psi_{s,s'}\right) d\log \boldsymbol{A} + \boldsymbol{\mathcal{E}}\; d\log \boldsymbol{X}}_{d\log \boldsymbol{\mathcal{N}}_{\boldsymbol{\mathcal{H}}}} \right\}' \boldsymbol{\mathcal{H}}\left(\Psi_{s,s'}\right) \tag{A.11}$$

where:

(*a*) $\boldsymbol{N}$ *identifies sectoral employment levels;*

...

(*f*) $\boldsymbol{\mathcal{H}}(\Psi_{s,s'}) = \left[\boldsymbol{I} - \Theta_{|\alpha}\, \boldsymbol{H}(\Psi_{s,s'})\right]^{-1}$ *is the Leontief inverse of the Input-Output matrix, which is as well adjusted by a compound of structural parameters now defined as* $\Theta_{|\alpha} = \frac{1}{1+\phi-\alpha}\,\alpha$.

▸ Go back, employment

Proposition 6 (Leontief propagation under factor input demand distance)

Consider an Input-Output economy defined by a labour market equilibrium as stated in eq. (A.7). Then, the direct and indirect network effects governing the response of sectoral employment is a first-order (log-linear) approximation given by

$$d\log \mathbf{N} = \Theta \left\{ d\log C + d\log \boldsymbol{\mathcal{S}} + d\log \boldsymbol{\mathcal{N}}_{\boldsymbol{\mathcal{H}}} + \underbrace{\boldsymbol{\mathcal{D}}\left[d\log \mathbf{N}\Big(\Phi(s)\Big) - d\log \mathbf{N}\right]}_{d\log \boldsymbol{\mathcal{D}}^{fd}(n)} \right\}' \boldsymbol{\mathcal{H}}\Big(\Psi_{s,s'}\Big) \qquad \text{(A.12)}$$

where:

(*a*) $\mathbf{N}$ *identifies sectoral employment levels;*

. . .

(*g*) $\boldsymbol{\mathcal{D}}$ *identifies all the network distances across any pair of sectors;*

(*h.i*) $\boldsymbol{\mathcal{D}}^{fd}(n)$ *identifies the production network distance effect of other sectors' changes in employment levels impacting sector-s, when these are buying their intermediate inputs from the same sector(s).*

Go back, demand distance

PROPOSITION 7 (Leontief propagation under factor input supply distance)

Consider an Input-Output economy defined by a labour market equilibrium as stated in eq. (A.8). Then, the direct and indirect network effects governing the response of sectoral employment is a first-order (log-linear) approximation given by

$$d\log \boldsymbol{N} = \Theta \left\{ d\log C + d\log \boldsymbol{\mathcal{S}} + d\log \boldsymbol{\mathcal{N}}_{\mathcal{H}} + \underbrace{\boldsymbol{\mathcal{D}} \left[d\log \boldsymbol{P}\Big(\Phi(s)\Big) - d\log \boldsymbol{P} \right]}_{d\log \boldsymbol{\mathcal{D}}^{fs}(p)} \right\}' \boldsymbol{\mathcal{H}}\Big(\Psi_{s,s'}\Big) \tag{A.13}$$

where:

(a) $\boldsymbol{N}$ *identifies sectoral employment levels;*

...

(g) $\boldsymbol{\mathcal{D}}$ *identifies all the network distances across any pair of sectors;*

(h.ii) $\boldsymbol{\mathcal{D}}^{fs}(p)$ *identifies the production network distance effect of other sectors' changes in employment levels impacting sector-s, when these are selling part of their output to the same sector(s).*

▸ Go back, supply distance

Theorem for distance matrix weighting

Theorem 2 (On the weighting of network "economic" distances)

In an Input-Output economy, the horizontal dimension of a network (defined by demand- and supply-driven distance matrices) is not weighted by its vertical dimension (defined by the direct or Leontief inverse matrix).

▸ Go back, distance matrix characteristics ▸ Go back, model discussion

Insights for aggregate business cycle

- Equilibrium adjustments in production networks are central to understand the origins of aggregate business cycles
- Fluctuations in aggregate real GDP {*e.g.*, vom Lehn and Winberry (2022)}:

$$d\log Y = \sum_{s\in\Phi(s)} \Big(\lambda(s)\; d\log z(s) + \nu(s)\; d\log n(s)\Big) \tag{A.14}$$

that is, under network distances,

$$d\log Y = \begin{cases} \sum_{s\in\Phi(s)} \Big(\lambda(s)\; d\log z(s) + \nu(s)\; \Delta\,\text{comovement}\Big) & \textit{under major distance} \\ \sum_{s\in\Phi(s)} \Big(\lambda(s)\; d\log z(s) + \nu(s)\; \Delta\,\text{reallocation}\Big) & \textit{under minor distance} \end{cases}$$

☞ different network distances generate different fluctuations in aggregate output

Go back

- ***market share, after redefinitions* Input-Output tables**
  - square matrix, $\underset{S\times S}{\boldsymbol{U}}$, whose diagonal correspond to sector-share of own production while:
    - ⋆ $\underset{S\times S}{\boldsymbol{U}^{up}} = uppertriangular(\boldsymbol{U})$ is the share of production of sector-s that is bought by sector-s';
    - ⋆ $\underset{S\times S}{\boldsymbol{U}^{dw}} = lowertriangular(\boldsymbol{U})$ is the share of production that sector-s buys from sector-s'.
  - in the dataset built, I compute $\boldsymbol{u} = \mathbf{1} - diag(\boldsymbol{U})$ to extract the *total share* of its total output that sector-s buys from other sectors, where $\mathbf{1}$ an $S \times 1$ vector of ones.
- ***commodity-by-commodity total direct requirements* Input-Output tables**
  - square matrix, $\underset{S\times S}{\boldsymbol{V}}$, in which each entry corresponds to:
    - ⋆ $\underset{S\times S}{\boldsymbol{V}^{up}} = uppertriangular(\boldsymbol{V})$ is the dollar value of one unit of sector-s that is bought by sector-s';
    - ⋆ $\underset{S\times S}{\boldsymbol{V}^{dw}} = lowertriangular(\boldsymbol{V})$ is the dollar value of sector-s in order to buy one unit from sector-s'.
  - in the dataset built, I compute $\sum_{row} \boldsymbol{V}$ to extract the *total network value* of sector-s.

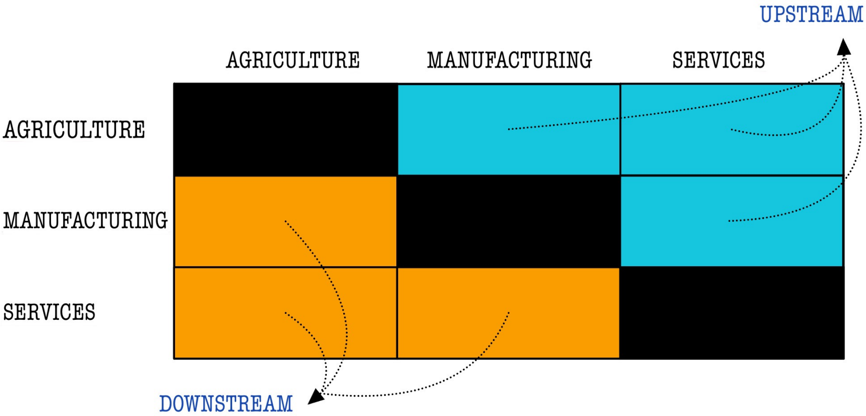
→ *row sectors* identify *supplier (upstream)*, and *column sectors* identify *buyer (downstream)*

Interpreting an Input-Output matrix, cells

	AGRICULTURE	MANUFACTURING	SERVICES
AGRICULTURE		AGRICULTURE **SELLS** TO MANUFACTURING	AGRICULTURE **SELLS** TO SERVICES
MANUFACTURING	AGRICULTURE **BUYS** FROM MANUFACTURING		MANUFACTURING **SELLS** TO SERVICES
SERVICES	AGRICULTURE **BUYS** FROM SERVICES	MANUFACTURING **BUYS** FROM SERVICES	

Go back

Interpreting an Input-Output matrix, types of sectors



Go back

- To investigate the network structure:
  - → upstream and downstream sectors, as well as sectoral distances, are computed through the *commodity-by-commodity direct requirements* table as of 2007;
  - → only to compute distances, disregard small transactions

$$h(s,s') = \frac{v(s,s')}{\tilde{v}(s)} \quad \textit{is set to} \quad 0 \quad \textit{if} \quad h(s,s') < 0.01$$

where $\tilde{v}(s) = \frac{v(s,s')}{\sum_{s' \in row} v(s,s')}$ is sector-s's total input sales $\left(\textit{i.e., row sum} \text{ of matrix } \underset{S\times S}{\boldsymbol{V}}\right)$.

- Define $\underset{S\times 1}{\tilde{V}} = \left[\tilde{v}(1), \ldots, \tilde{v}(s), \ldots, \tilde{v}(S)\right]'$. So, the considered Input-Output structure is

$$\underset{S\times S}{\boldsymbol{H}} = \underset{S\times S}{\boldsymbol{V}}{}' \underset{S\times 1}{\tilde{V}}$$

in which each cell, $h(s,s')$, is greater than 1%; analogously under a *column sum*

Go back

- Exposure to inter-sectoral trade can be twofold:

  ① *direct*, which accounts for the exposure of sector-s to its supply chain;

  → however, also a certain intermediate input is the outcome of a separated supply chain. As a result, since sector-s buys intermediate inputs from other sectors, it is exposed also to the supply chain of each sector $s' \neq s$. Therefore . . .

  ② *indirect*, which accounts for the exposure of sector-s to its supply chain, and to that of its trading-partner sectors. For instance, all intermediate inputs of sector-s, $X(s)$, are given by:

$$X(s) = \underbrace{h(s,s)\,x(s,s) + h(s,s')\,x(s,s') + \cdots + h(s,S)\,x(s,S)}_{direct}$$

$$= \underbrace{h(s,s)\,x(s,s)}_{direct} + \underbrace{h(s,s')\,h(s',s)x(s,s) + h(s,s')\,h(s',s')\,x(s',s') + \cdots + h(s,s')\,h(s',S)\,x(S,S)}_{indirect,\ s'}$$

$$+ \cdots + \underbrace{h(s,S)\,h(S,s)\,x(s,s) + h(s,S)\,h(s,s')\,x(s',s') + \cdots + h(s,S)\,h(S,S)\,x(S,S)}_{indirect,\ S}$$

→ which, in matrix form, is $\underset{S\times 1}{\boldsymbol{X}} = \underset{S\times S}{\boldsymbol{H}}{}' \underset{S\times 1}{\boldsymbol{X}}$, so to have that $\underbrace{\left(\underset{S\times S}{\boldsymbol{I}} - \underset{S\times S}{\boldsymbol{H}}\right)^{-1}}_{\widehat{\boldsymbol{H}},\ \textit{Leontief inverse}\text{ of matrix }\boldsymbol{H}}{}' \underset{S\times 1}{\boldsymbol{X}} = \boldsymbol{0}$

Go back

Network representation

share of sectoral total sales

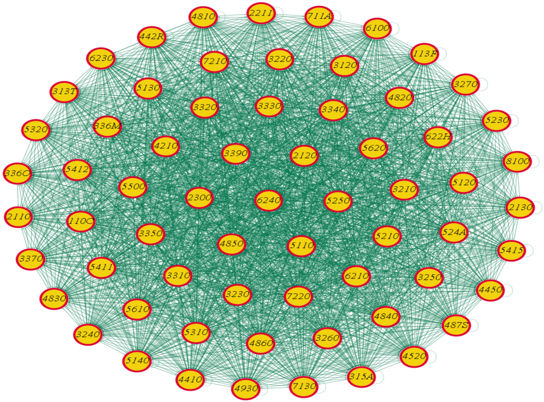


Figure B.1: Production network of the U.S. economy in 2007

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Network representation

share of sectoral total purchases

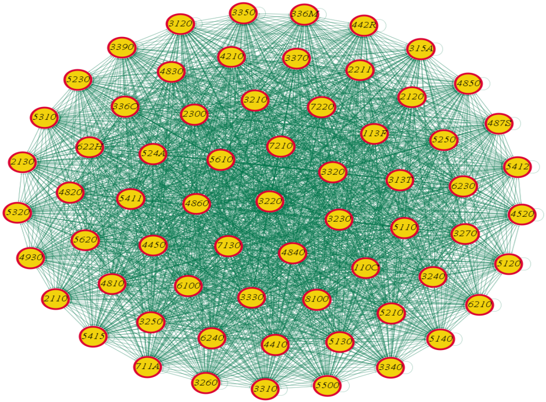

Figure B.2: Production network of the U.S. economy in 2007

Go back

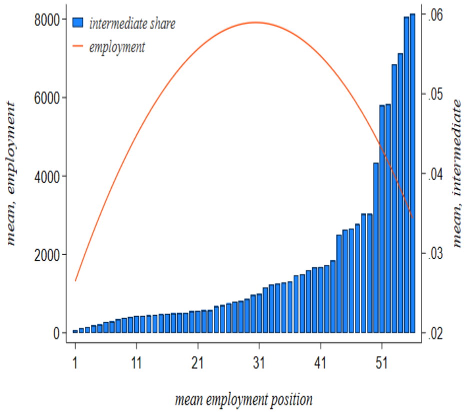

Figure B.3: Levels

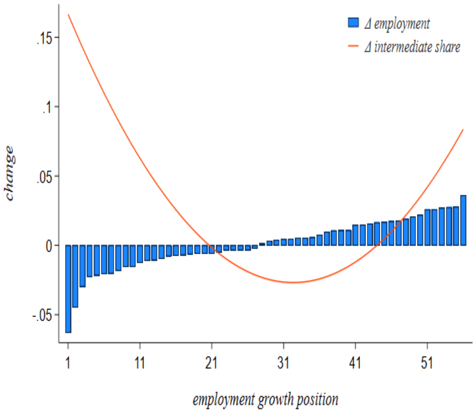

Figure B.4: Changes

Shortest path algorithm.

Central idea is to find, for each single node, all the other nodes that can be reached in one step, then in two steps, and so on. Each time a node is reached for the first time, it records the number of steps it took to get there – this is the shortest path approach. In other words, find the shortest number of steps it takes to get from every node to every other node in a network. In particular:

(*a*) identify the direct connections between any pair of nodes (those that take exactly one step to be connected);

(*b*) for not-connected pairs, increase the path length by 1 in each round (two steps, then three steps, etc.). Multiply the graph by itself to discover which new pairs of nodes are now connected through longer paths;

(*c*) if a pair of nodes becomes connected (*i.e.*, only if it has not be already found a shorter path), record the current length as the shortest distance between them;

(*d*) repeat this process until any new reachable pairs of nodes is found. For any pair of nodes that are still unreachable (*i.e.*, the distance is still zero and they are not linked in the network), set their distance to infinity.

□

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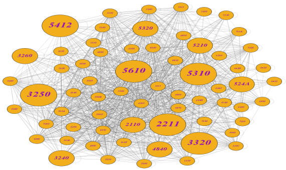
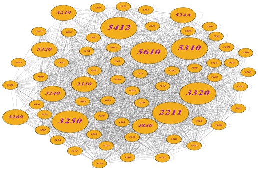
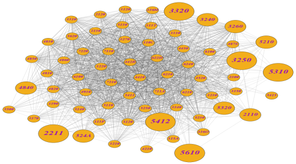


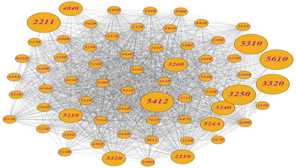
(a) Demand, $d = 1$



(b) Supply, $d = 1$



(c) Demand, $d = 2$



(d) Supply, $d = 2$

Figure B.5: Visualization of production network distances

Go back

Characteristics of network-based distances, extensive margin

(1) The distribution of horizontal complementarities is uneven

- the factor input demand network is slightly denser than the supply network ($d = 1$), but then this pattern reverses ($d = 2$)

(2) Demand-driven linkages tend to be concentrated within a subset of sectors

- over 35% of sectors do not exhibit soft demand-based connections

(3) Sectors generally share the same set of downstream buyers but not the same set of upstream suppliers

- this difference becomes evident at greater distances ($d = 2$)

(4) Sectors with the highest number of production network connections tend to be evenly central at shorter distances

Go back

Intensive margin values for network distance

- As outlined in Conley and Dupor (2003), two measures of *network distance* can be computed. Labelling the set of sectors with $\Phi(s)$, a pair of sectors (s, s') is related as:

① ***factor input demand***, when buying similar inputs. Given $\widehat{h}(s, s') = \frac{v(s,s')}{\sum_{s' \in row} v(s,s')}$, then

$$\mathcal{D}^{fd} = \left\{ \sum_{k \in \Phi(s)} \left[\widehat{h}(k, s) - \widehat{h}(k, s') \right]^2 \right\}^{\frac{1}{2}}$$

is measuring *proximity* with respect to production function requirements. *see Figure B.6*

② ***factor input supply***, when selling to similar sectors. Given $h(s, s') = \frac{v(s,s')}{\sum_{s' \in column} v(s,s')}$, then

$$\mathcal{D}^{fs} = \left\{ \sum_{k \in \Phi(s)} \left[h(s, k) - h(s', k) \right]^2 \right\}^{\frac{1}{2}}$$

is measuring *comovement* of sectoral productivity growth; *see Figure B.7*

- Distances are computed as those from the *directed network*

Sectoral distance in the network, factor input demand

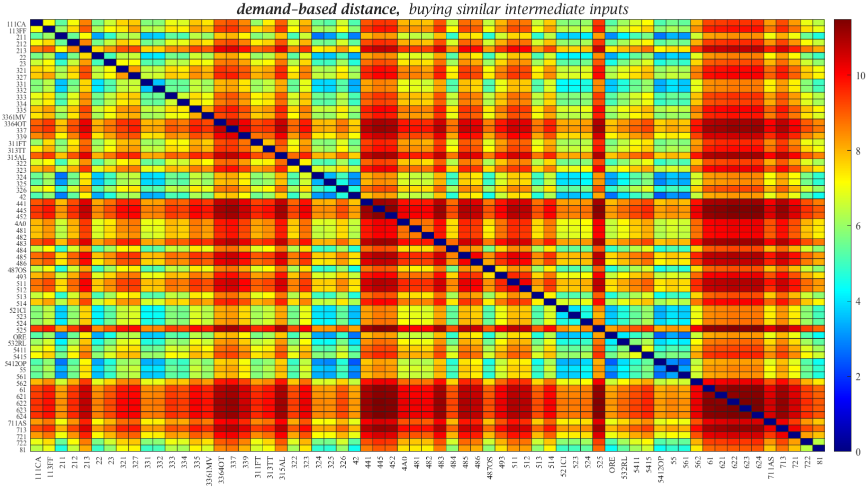


Figure B.6: *Factor demand distances* in the Input-Output matrix, U.S. 2007

Go back

Sectoral distance in the network, factor input supply

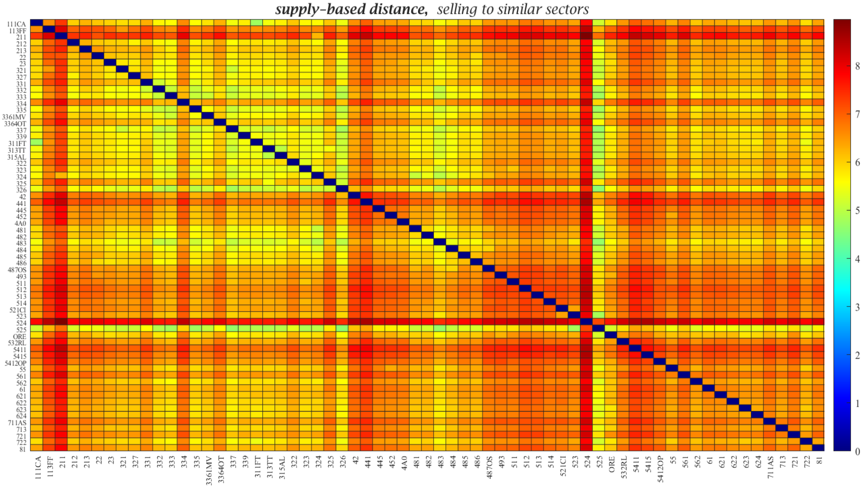
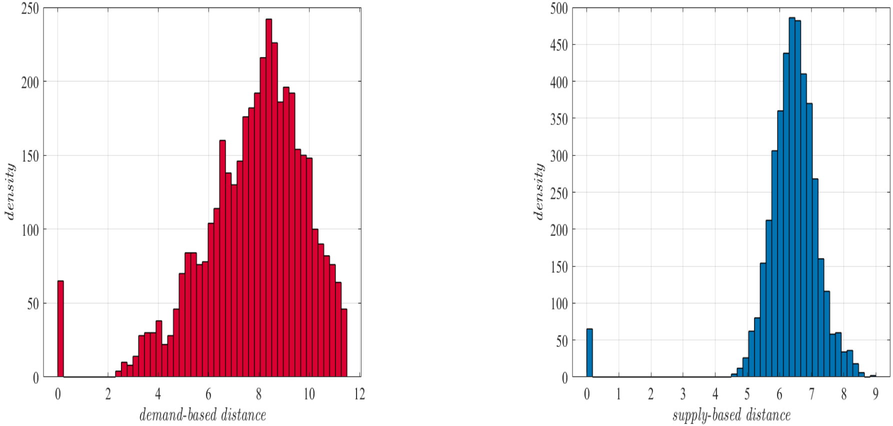


Figure B.7: *Factor supply distances* in the Input-Output matrix, U.S. 2007

Go back

Distribution of intensive margin network distances



(a) Factor input demand

(b) Factor input supply

Figure B.8: Distribution of network economic distances

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Correlation between intensive margin distances

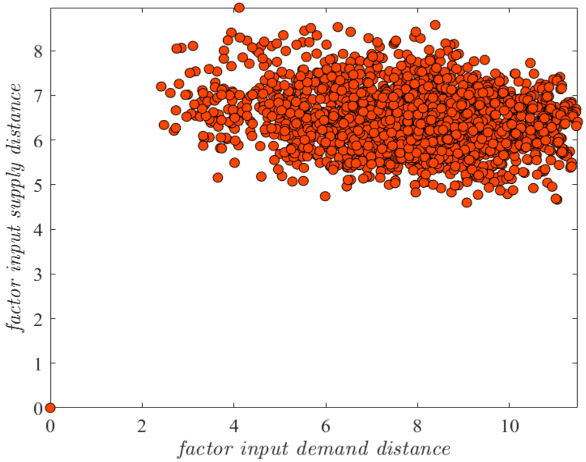
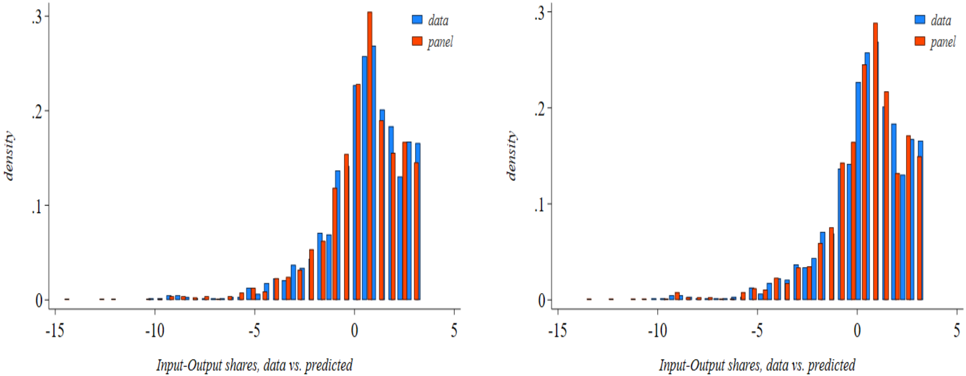


Figure B.9: Correlation of network economic distances

Go back

Distribution of estimated values from residuals, I-O shares



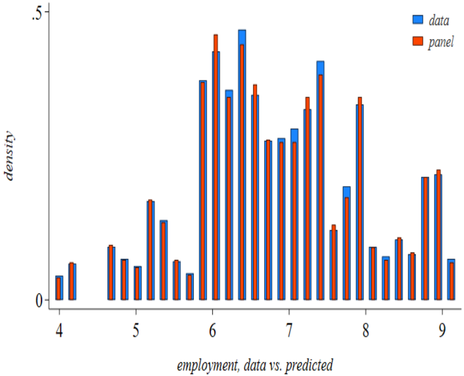
(a) Factor input demand

(b) Factor input supply

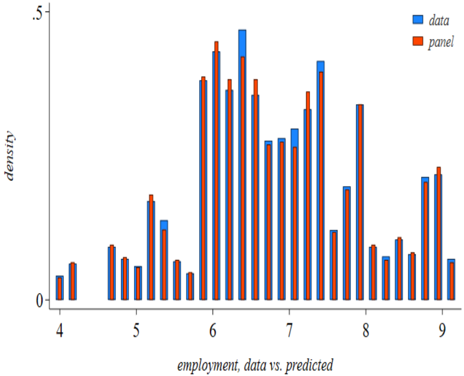
Figure B.10: Distribution for sectoral intermediate inputs, *log*-values

Go back

Distribution of estimated values from residuals, employment



(a) Factor input demand

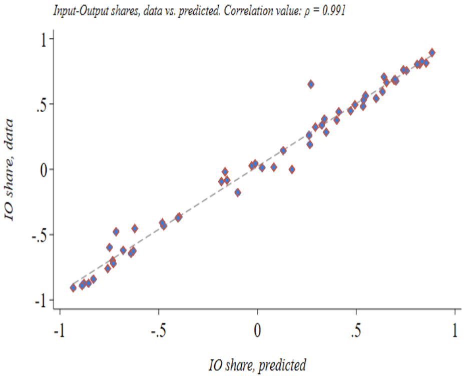


(b) Factor input supply

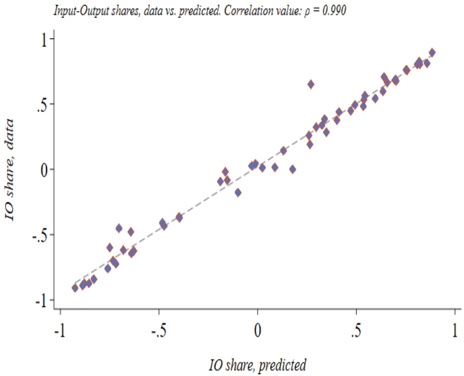
Figure B.11: Distribution for sectoral employment, *log*-values

Go back

Correlation of estimated values from residuals, I-O shares



(a) Factor input demand

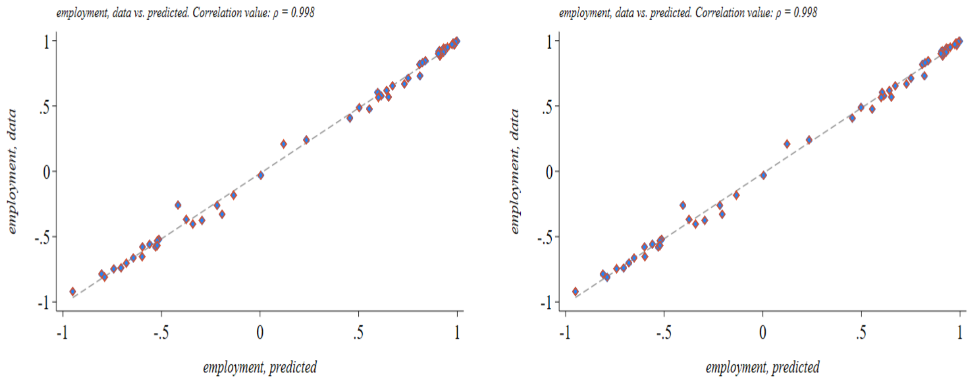


(b) Factor input supply

Figure B.12: Correlation of sectoral intermediate inputs, *log*-values

Go back

Correlation of estimated values from residuals, employment



(a) Factor input demand **(b)** Factor input supply

Figure B.13: Correlation of sectoral employment, *log*-values

Go back

Defining independent variables

- employment growth, $\dot{n}_t(s) = n_t(s) - n_{t-1}(s)$;
- network distance-d computed as a *shortest path problem*;
- given Φ_s^j the set of sectors which are $j \in \{upstream, downstream\}$ to sector-s:

  (a) upstream employment growth, $\dot{n}_t\left(\Phi_s^{up}\right) = \sum_{s^{up} \neq s} \left[\ell(s, s^{up})\left(n_t(s^{up}) - n_{t-1}(s^{up})\right)\right]$;

  (b) downstream employment growth, $\dot{n}_t\left(\Phi_s^{dw}\right) = \sum_{s^{dw} \neq s} \left[\ell(s, s^{dw})\left(n_t(s^{dw}) - n_{t-1}(s^{dw})\right)\right]$;

  where $\ell(s, s^j) = \widehat{h}(s, s^j)$ is the Leontief weight of sector s^j on sector-s.

- controls:

  (a) sectoral employment size $(n(s))$, value-added $(y(s))$ and its growth rate, and net exports;

  (b) aggregate employment size (N) and value-added (Y), and their growth rates.

Go back, I-O shares | Go back, employment | Go back, employment (separated)

EMPIRICAL FRAMEWORK: IDENTIFICATION STEP

① Purge the dynamics of sectoral I-O shares from movements in sectoral employment

- panel Fixed-Effects (FE) regression over $s \in \Phi(s)$ sectors

$$x_t(s) = \underbrace{\beta_n\ \dot{n}_t(s)}_{\textit{emp. growth}} + \underbrace{\sum_d \beta_{n(d)}\ \dot{n}_t(\Phi_s, d)}_{\textit{emp. growth, network distance}} + \underbrace{\sum_k \beta_{n(k)}\ \dot{n}_t(\Phi_s, k)}_{\textit{emp. growth, upstream and downstream}}$$

$$+ \underbrace{\beta_{z(s)}\ \mathcal{Z}_t(s) + \beta_{\dot{z}(s)}\ \dot{\mathcal{Z}}_t(s)}_{\textit{sector-level controls}} + \underbrace{\beta_z\ \mathcal{Z}_t + \beta_{\dot{z}}\ \dot{\mathcal{Z}}_t}_{\textit{aggregate controls}} + \underbrace{\psi(s)}_{\textit{sector Fe}} + u_t^x(s)$$

- estimated residuals $\left(\widehat{u}_t^x(s)\right)$ identify the residual variation in I-O shares

▸ Variables' definition ▸ Estimated I-O residuals, distribution

Empirical framework: dynamic step

② Estimate panel Local Projections (Lp) to study the dynamic effects of changes in sectoral I-O intermediate shares on its (cumulative) employment

- given $\hbar = 5$ horizons, sequence of panel predictive regressions of the form

$$\dot{n}^{cum}_{t+\hbar}(s) = \beta_{\widehat{u}^x,\hbar}\, \widehat{u}^x_t(s) \;+\; \underbrace{\psi(t+\hbar) \;+\; \psi_\hbar(s)}_{\textit{horizon and sector Fe}} \;+ \nu^x_{t+\hbar}(s)$$

. . . also considering d^{th}-distance across sectors

$$\dot{n}^{cum}_{t+\hbar}(s) = \beta_{\widehat{u}^x(d),\hbar} \sum_{s' \neq s} \widehat{u}^x_t(s', d) \;+\; \underbrace{\psi(t+\hbar) \;+\; \psi_\hbar(s)}_{\textit{horizon and sector Fe}} \;+ \nu^x_{t+\hbar}(s, d)$$

- variables at $\hbar$ identifies the cumulative difference between time t and $t+\hbar$

Network shares and employment

factor input demand and directed network

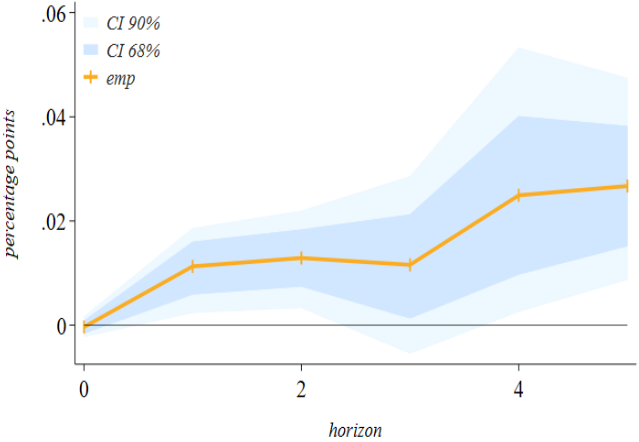
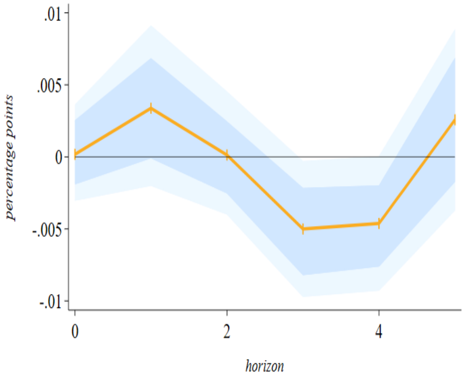


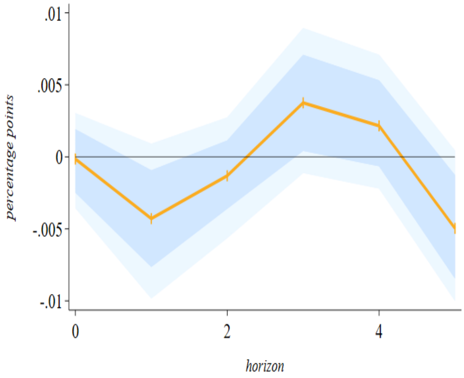
Figure D.1: Sectoral employment response following a 1% increase in own I-O share

Network shares and employment, distance

factor input demand and directed network



(a) $d = 1$



(b) $d = 2$

Figure D.2: Sectoral employment response following a 1% increase in I-O share of nearby sectors

Network shares and employment

factor input supply and directed network

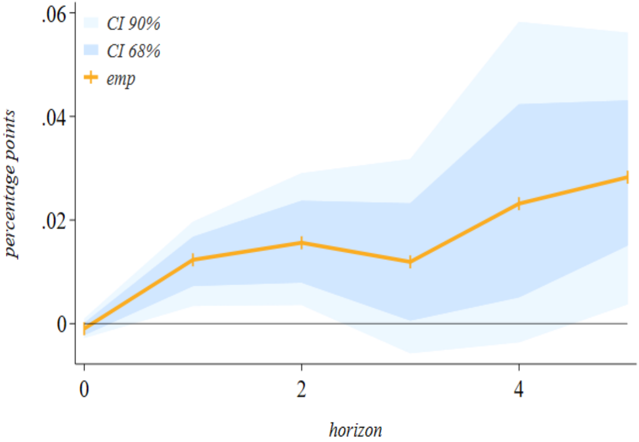
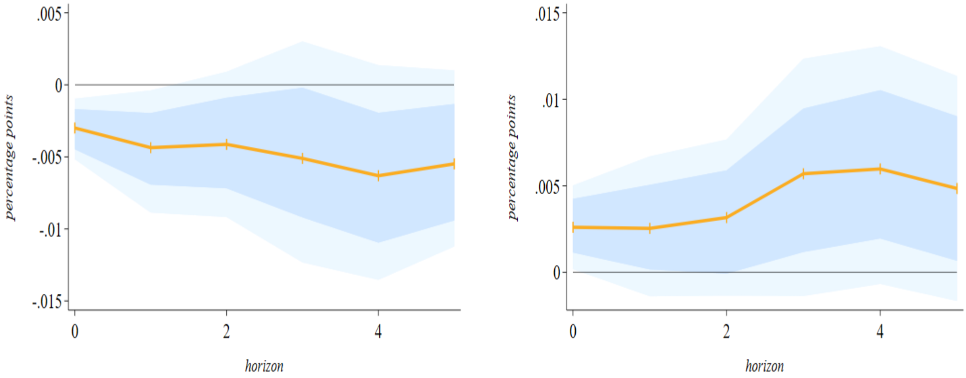


Figure D.3: Sectoral employment response following a 1% increase in own I-O share

Network shares and employment, distance

factor input supply and directed network



(a) $d = 1$

(b) $d = 2$

Figure D.4: Sectoral employment response following a 1% increase in I-O share of nearby sectors

① Purge the dynamics of sectoral employment from movements in sectoral output

- panel Fixed-Effects (Fe) regression over $s \in \Phi(s)$ sectors

$$n_t(s) = \underbrace{\beta_y \, \dot{y}_t(s)}_{\text{output growth}} + \underbrace{\sum_d \beta_{y(d)} \, \dot{y}_t(\Phi_s, d)}_{\text{output growth, network distance}} + \underbrace{\sum_{k \in \{up,dw\}} \beta_{y(j)} \, \dot{y}_t(\Phi_s, k)}_{\text{output growth, upstream and downstream}}$$

$$+ \underbrace{\beta_{z(s)} \, \mathcal{Z}_t(s) + \beta_{\dot{Z}(s)} \, \dot{\mathcal{Z}}_t(s)}_{\text{sector-level controls}} + \underbrace{\beta_z \, \mathcal{Z}_t + \beta_{\dot{Z}} \, \dot{\mathcal{Z}}_t}_{\text{aggregate controls}} + \underbrace{\psi(s)}_{\text{sector Fe}} + u_t^n(s)$$

- estimated residuals $\left(\widehat{u}_t^n(s)\right)$ identify the residual variation in employment

▸ Variables' definition

Empirical framework: dynamic step

② Estimate panel Local Projections (LP) to study the dynamic effects of changes in sectoral employment of nearby sectors on its (cumulative) employment

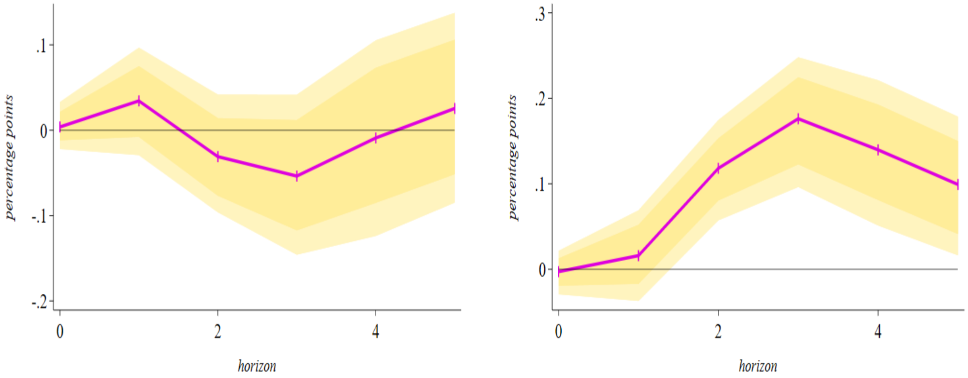
- given $\hbar = 5$ horizons and d^{th}-distance, sequence of panel predictive regressions

$$\dot{n}^{cum}_{t+\hbar}(s) = \beta_{\hat{n}(d),\hbar} \sum_{s' \neq s} \hat{u}^{n}_{t}(s', d) \; + \; \underbrace{\psi(t+\hbar) \; + \; \psi_{\hbar}(s)}_{horizon\ and\ sector\ Fe} \; + \nu^{n}_{t+\hbar}(s, d)$$

- variables at $\hbar$ identifies the cumulative difference between time t and $t + \hbar$

Employment effects, distance

factor input demand and directed network



(a) $d = 1$

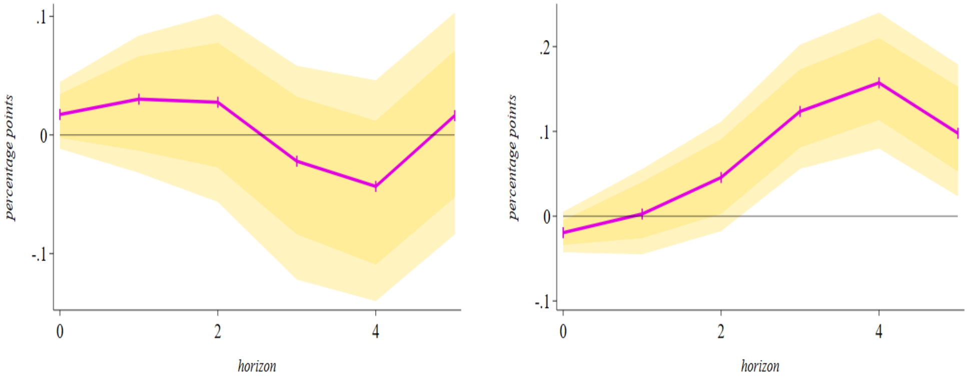
(b) $d = 2$

Figure E.1: Employment comovement under direct demand linkages

▸ Go back

Employment effects, distance

factor input supply and directed network



(a) $d = 1$ **(b)** $d = 2$

Figure E.2: Employment comovement under direct supply linkages

Go back